Repeated games

- This is a special class of games in extensive form
- An identical „stage game” is repeated several times
- The payoffs are paid out after each „stage game”
- The payoffs from future period are discounted (the payoff u in period t is worth $\delta^t u$ now)

Repeated games - example

- This stage game is played 2 times

		Player 2			
		L	M	R	
	L	1, 1	0, 0	5, 0	
Player 1	M	0, 0	3, 3	0, 0	
	R	0, 5	0, 0	4, 4	

- Any SPNE where players behave differently than in a 1-time game?

Example cont.

- Only (L,L) and (M,M) are pure-strategy NE of the stage game
- Therefore (R, R) cannot be played in a SPNE in stage 2
- But it can be played in stage 1 of a SPNE! Consider the following strategy for both players:
  - play R in stage 1;
  - if (R,R) was played in stage 1 (cooperation), play M in stage 2 (the good NE);
  - if (R,R) was **not** played in stage 1 (betrayal), play L in stage 2 (the bad NE).
- This constitutes a SPNE. Deviating in stage 2 is not profitable, because a pair of the strategies constitutes a NE after any history. Deviating to L in stage 1 is not profitable, because you'd gain 1 but loose $\delta 2$ in stage 2.

Finitely repeated games

- Consider a finitely repeated version of the advertising game

		Player 2	
		A	N
Player 1	A	40, 40	60, 30
	N	30, 60	50, 50

- In this game, no punishment is available
- The only SPNE is for both players to choose A in every stage of the game, even if the game is repeated very many (but not infinitely many) times

In(de)finitely repeated games

- If a game like the Ad game is repeated **infinitely** many times, then the possibility of future punishment exists in every subgame
- The same is true if the game is repeated **indefinitely** many times, (as long as it is potentially infinite)
- Repeated games that go on forever (or potentially forever) are called **supergames**

Cooperative equilibria

- We will use the infinitely-repeated Bertrand model to illustrate how cooperation can emerge as an SPNE
- Consider the following strategy (called ‘grim trigger strategy’):
  - Begin the game by setting p^M in stage 1
  - Continue to set p^M, if you did not observe any other prices set in the past
  - Set $p = c$ if you’ve observed at least one of the players set the price different than p^M at least once in the past
  - In other words, begin with cooperation and continue cooperating, but switch to marginal-cost pricing as soon as you observe non-cooperative price by any player, and never go back to monopoly price

SPNE

- We will now show that a pair of grim trigger strategies constitutes an SPNE
- To do that, we will use the ‘one-shot deviation’ criterion: in a repeated game, if a one-time deviation is unprofitable, multiple deviations cannot be profitable either. So it is enough to check one-time deviations
- There are two types of deviations:
  - Deviation during ‘cooperation phase’
  - Deviation during ‘punishment phase’

SPNE cont.

- A deviation during punishment cannot be profitable, since players choose actions that form a NE in the stage game
- A deviation during cooperation is not profitable if the profit from continuing cooperation exceeds the profit from deviation

$$\frac{\pi^m}{2} + \delta \frac{\pi^m}{2} + \delta^2 \frac{\pi^m}{2} + \ldots \geq \pi^m + \delta 0 + \delta^2 0 + \ldots$$

$$\frac{\pi^m}{2} \frac{1}{1-\delta} \geq \pi^m$$

$$\delta \geq \tfrac{1}{2}$$

Tacit collusion

- Hence firms can sustain perfect collusion if they are patient enough (if δ is close enough to 1)
- This type of cooperation by firm, which is sustained by the threat of future retaliation, is called 'Tacit collusion' (silent collusion)

Folk theorem

- Notice that cooperation can be supported by other strategies (i.e. trigger strategies with finite punishment periods)
- Notice also that playing *c* in every stage is also an SPNE
- What's more it can be shown that almost anything can emerge an SPNE outcome in this type of repeated games
- A Folk Theorem:Any pair of profits of firm 1 and 2 $(\pi_{1,}\ \pi_2)$ such that $\pi_1 + \pi_2 \leq \pi^m$ can be an equilibrium average per-period payoff in a Bertrand supergame for δ sufficiently close to 1.
- However, we typically apply Pareto Perfection, as in the following application

Application – Rottemberg-Saloner model

- 2 firms compete as in the Bertrand supergame, but demand is stochastic
- In state 1 the demand is low, in state 2 the demand is high. Both states are equally probable.
- Full collusion (Pareto Perfect SPNE) is reached when monopoly prices (p_1^M, p_2^M) are charged by both firms in both states
- Assume grim trigger strategies

Application – Rottemberg-Saloner model

- The fully collusive profit is

$$V = \tfrac{1}{2}\left(\frac{\Pi_1^M}{2} + \frac{\Pi_2^M}{2}\right)/(1-\delta)$$

- Deviating is more profitable in state 2, but will be worse than collusion if

$$\frac{\Pi_2^M}{2} + \delta V \geq \Pi_2^M$$

- Substituting for V and solving yields

$$\delta \geq 2\Big/\left(\frac{\Pi_1^M}{\Pi_2^M} + 3\right) \equiv \delta_0$$

Rottemberg-Saloner model

- So if $\delta \geq \delta_0$ full collusion is possible
- We also know that if $\delta < ½$ then no collusion is possible
- It can be shown, that when $½ < \delta \leq \delta_0$, then we will have ‘imperfect’ collusion, where firms set prices below monopoly level in high-demand states, in order to sustain cooperation

Rottemberg-Saloner model

- For ½ < $\delta \leq \delta_0$ firms will try to set the highest sustainable price (Pareto perfect SPNE). They will maximize:

$$V = \left(\frac{\Pi_1(p_1)}{4} + \frac{\Pi_2(p_2)}{4} \right) / (1-\delta)$$

- Subject to 2 constraints

$$\delta \left(\frac{\Pi_1(p_1)}{4} + \frac{\Pi_2(p_2)}{4} \right) / (1-\delta) \geq \frac{\Pi_1(p_1)}{2}$$

$$\delta \left(\frac{\Pi_1(p_1)}{4} + \frac{\Pi_2(p_2)}{4} \right) / (1-\delta) \geq \frac{\Pi_2(p_1)}{2}$$

Rottemberg-Saloner model

- Which is equivalent to maximizing

$$\max \Pi_1(p_1) + \Pi_2(p_2)$$
$$s.t.$$
$$\Pi_1(p_1) \leq K\Pi_2(p_2)$$
$$\Pi_2(p_2) \leq K\Pi_1(p_1)$$

- Where $K \geq 1$ is a constant
- We have 2 cases:
  - Case 1: Both constraints are binding
  - Case 2: Only the second constraint is binding

Rottemberg-Saloner model

- Case 1: $\Pi_2(p_2) = \Pi_1(p_1)$, but

max $\Pi_2(p_2) >$ max $\Pi_1(p_1)$,

so $p_1 = p_1^M$ and $p_2 < p_1^M$
(price wars during booms)

- Case 2: $\Pi_2(p_2) = K\Pi_1(p_1)$,

so $p_1 = p_1^M$ and $p_2 < p_2^M$

- So we should observe ‘price-wars’ during economic boom periods